Antenna Phase Center Effects and Measurements in GNSS Ranging Applications

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Abstract—Antenna phase center offset (PCO) location and phase center variation (PCV) play an important role in high-precision GNSS ranging applications that use carrier-phase measurements. For ranging systems that use CDMA code-phase measurements, other statistics play a similar role – antenna group delay center offset (GDCO) and group delay variation (GDV). This paper describes the methods of how to measure and compute these statistics based on amplitude and phase pattern measured for a given antenna. A susceptibility analysis of these statistics versus multipath and antenna design parameters is also made. Practical examples on how to deal with undesired movement of PCO or larger variation of PCV is presented as well.

Keywords- phase center; group delay center; multipath; GNSS antenna; phase center offset (PCO); phase center variation (PCV)

I. INTRODUCTION

The electrical location of mean phase center offset (PCO) of an antenna plays an important role in many modern scientific and engineering applications. It matters in such applications as ranging (positioning, attitude, surveying, etc.), imaging applications (radio telescopes, image radars), antenna array (i.e. null steering), ultra wideband (UWB) with signal cohesion and group delay distortion caused by phase center variations (PCV). This paper addresses the subject of phase center determination for GNSS ranging applications.

In the following section, the subject is extended to group delay center location and its impact on the ranging systems.

The topic of multipath is then introduced using two-ray generic model for any type of smooth reflective surface (perfect electric conductors, soil, water, concrete, etc.). The effect of multipath on PCO/PCV degradation is shown for a typical high grade survey antenna.

A relatively simple solution to mitigate multipath is shown in section VII using a dual circularly polarized antenna.

II. PHASE CENTER DEFINITION

Phase center is defined in the IEEE standards as: “The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the phase of a given field component over the surface of the radiation sphere is “essentially” constant, at least over the portion of the surface where the radiation is significant”.

Since nothing is perfect, this “real” measured phase sphere will have deviations from an “ideal” phase sphere centered on the mean phase of the antenna as shown in Figure 1.

Three terms that are important for GNSS ranging applications are PCO (phase center offset), PCV (phase center variation), and ARP (antenna reference point).

PCO is the mean location of the antenna phase center which normally does not coincide with ARP (a visible outside reference point on an antenna chassis). Typical antenna installation for surveying is made using the ARP point, therefore a correction must be made to translate this point to PCO and vice-versa. Any signal received from a given satellite in direction \( \mathbf{r}_0 \) will experience a phase shift caused by PCV. For millimetre (mm) ranging applications, such shift has to be compensated for, in order to achieve precise (sub-millimetre) electrical position of the antenna. Such correction is computed using (1):

\[
S_{\text{ARP}} = r + PCO \cdot r_0 + PCV(\phi, \theta) + \varepsilon \tag{1}
\]
where \(\phi, \theta\) are azimuth and elevation angles towards the signal source. PCO is found by minimizing the following cost function:

\[
\sum (PCV)^2 = \text{Min}
\]


### III. PHASE CENTER DETERMINATION

There are several papers published on the subject of phase center determination. They can be classified into: Second Derivative Method [3], Two Point Method [4], Edge Diffraction Method [5], Differential Phase Method [6], Three Antenna Method [7], Time of Arrival (TOA) Method [9], and Spherical Phase Method (this paper).

The intent of this paper is to present a simple and practical method of measuring and computing phase center based on real antenna measurements (i.e. from an anechoic chamber). We will use a least squares method and start with the equation of spherical phase expansion from its origin located at point PCO \((x,y,z)\) as shown in the equation below. A term “c” is added that can represent some fixed constant. Azimuth and elevation angles are denoted as \(\phi\) and \(\theta\), while wavelength is \(\lambda\).

\[
\text{Phase}(\phi, \theta) = \frac{2\pi}{\lambda} \left( x \cdot \cos \phi \sin \theta + y \cdot \sin \phi \sin \theta + z \cdot \cos \theta \right) + c
\]

or in matrix form:

\[
F(\phi, \theta) = \frac{2\pi}{\lambda} \begin{bmatrix} \cos \phi \sin \theta & \sin \phi \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ c \end{bmatrix}
\]

\[
= \Rightarrow \quad F = M \cdot P \quad (2b)
\]

To find phase center \(P\) we use eq. (2b), hence:

\[
P = M^{-1} \cdot F
\]

We are now ready to assemble a solution matrix using measurements obtained from sampling the phase pattern of the antenna to obtain a mean value of PCO.

One can add more columns on the right term to represents measurements made at several frequencies and correspondingly we will get mean PCO solution for all listed frequencies in one step using (3). Once the mean PCO is determined, we can multiply back by the spherical phase expansion matrix to obtain an ideal spherical phase pattern anchored at mean PCO determined from real measured phase pattern. The difference residual between the ideal and measured phase pattern will be simply phase center variation (PCV).

### IV. GROUP DELAY CENTER

GNSS systems can compute position two ways: using code measurements and/or carrier measurements. To increase carrier accuracy measurements, its PCO and PCV values must be corrected. Similarly for code measurements, equivalent group delay center offset (GDCO) and group delay variation (GDV) should be determined.

This is a different type of group delay notion normally perceived by the industry. It’s a spatially varying group delay initiated by the antenna pattern that will introduce a different error towards each satellite. It will mask itself as an additional ionospheric/tropospheric error. The group delay introduced by the receiver is basically constant and can be easily removed as part of the clock bias. This is not the case with spatial varying group delay introduced by an antenna.

A similar approach can be taken to compute the mean group delay center of an antenna as we did previously for carrier phase measurements in (3). We can replace phase measurements on the right term with phase derivative with respect to frequency at every angle \(\theta\) and \(\phi\).

\[
gd(\phi, \theta) = \frac{d \phi(\phi, \theta)}{d\lambda} \quad (4)
\]

Where \(d\lambda\) is the frequency bandwidth of the spreading code used for a particular GNSS system (i.e. 1.023 MHz for the L1 C/A code in the GPS system).

### V. MULTIPATH

The simplest multipath model is achieved using a two-ray model [11], where multipath is introduced by a single, specular reflection from the ground beneath the antenna. We can assume that ground is perfectly smooth for now, infinitely large and composed of homogeneous material. Since GNSS systems use circular polarization, we need to decompose the incoming signal into two linear vectors (E and H) and compute the reflection coefficient in both modes (TE and TM) using Fresnel reflection coefficients [12]. We express the reflected and transmitted fields as a function of incident fields which are combination of linearly independent polarizations.

\[
\Gamma^{TE} = \frac{\eta_2 \cos \phi' - \eta_1 \cos \phi}{\eta_2 \cos \phi' + \eta_1 \cos \phi}
\]

\[
\Gamma^{TM} = \frac{\eta_2 \cos \phi' - \eta_1 \cos \phi'}{\eta_2 \cos \phi' + \eta_1 \cos \phi'}
\]
Where \( \eta \) is a refractive index \( \eta = \sqrt{\mu/\varepsilon} \). Samples of curves are computed for sea water, dry sand and concrete. See Figures 2 and 3.

At the incident angle normal to the ground surface, both reflection modes (TE and TM) have equal amplitude and the same phase. The reflected wave has an opposite circular polarization as compared to the incident angle. For the lower elevation angle, the amplitude between the two modes diverge; however, they remain in phase.

The above paragraph refers to all ground types except for PEC (perfect electric conductor). In the PEC case, the reflected wave always has the opposite circular sense of polarization.

To keep the math simple we will now follow notation using Jones calculus [9][10]. In order to obtain a total response at the antenna terminal, the direct incident signal is multiplied by the antenna complex conjugate response and then added to its reflected replica multiplied by the antenna complex conjugate response at negative angle \( \theta \) (as shown in (6)).

\[
\hat{S}(\varphi, \theta) = \hat{C}^*(\varphi, \theta) \cdot \hat{E} + \hat{C}^*(\varphi,-\theta) \cdot \Gamma \cdot \hat{E} \cdot e^{j2\pi d/\lambda} \tag{6}
\]

where:
\[
\hat{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix}, \quad \hat{C}^*(\varphi, \theta) = \frac{a_{\theta,\varphi}}{\sqrt{2}} (1 \pm j) e^{j\varphi}\theta, \quad \hat{C}^*(\varphi,-\theta) = \frac{a_{\theta,\varphi}}{\sqrt{2}} (1 \pm j) e^{j\varphi}\theta.
\]

Distance \( d \) is the extra path length that the reflected ray has to travel compared to the incident ray [11]. It can be easily shown as:

\[
d = 2 \cdot h \sin(\theta)
\]

where \( h \) is height of the antenna PCO above the reflected plane.

\( \hat{C}^*(\varphi, \theta) \) either represents the complex conjugate of the antenna amplitude or the phase pattern using the Jones vector notation. Note that for negative angles of \( \theta \), this response is RHCP (right hand circularly polarized) or LHCP (left hand circularly polarized) antenna pattern. This is represented as the \( \pm j \) term. The sign depends on whether the angle of incidence is above (negative sign) or below (positive sign) the Brewster angle. The term \( \pm j \) vanishes to zero exactly at the Brewster angle, since there is no reflection for \( \Gamma^{TM} \). For PEC, this term is always negative, since the reflected wave has an opposite polarization as the incoming wave. This is not the case for any other material used as the reflected surface, where the polarization changes with the \( \theta \) angle and relation with respect to the Brewster angle.

Zero terms in the \( \Gamma \) matrix term represent the fact that we use a linear system placed in an orthogonal co-ordinate system.

### VI. EXAMPLES

This section contains an example of the multipath effect on a real antenna PCO location with ground consisting of wet sand as opposed to no reflections at all. A high performance survey grade antenna is used for this example. It is assumed that ground is infinitely large and symmetrical in the azimuthal (\( \varphi \)) direction. It is rarely the case that there will be such a perfectly symmetrical ground in the real world; therefore, the computed PCO movement (shown in Fig. 5) is smaller than anticipated due to the symmetry of the model used. Multipath effects are symmetrical in this model. Nevertheless, a
A larger effect can be observed on antenna amplitude pattern and PCO solution along z-axis (antenna height) under reflection conditions. Lack of space prevents us from showing these examples.

VII. MULTIPATH MITIGATION

One way to mitigate multipath is to have a dual linearly polarized antenna that has identical amplitude patterns. Combining two linearly polarized signals with ±90° phase gradient creates RHCP and LHCP polarization of the antenna. Using equation (6) we can compute the received signal for the RHCP antenna (first equation of (7)) and the LHCP antenna (second equation of (7)) as:

\[
\begin{align*}
S_{RHCP} &= a_R(\theta)e^{j(\delta(-\theta) - \phi(\theta))} + \frac{a_{R(-\theta)}e^{j(2\pi d/\lambda + \delta(-\theta))}}{2} \left( \Gamma_{TE} \mp \Gamma_{TM} \right) \\
S_{LHCP} &= 0 + \frac{a_{L(-\theta)}e^{j(2\pi d/\lambda + \delta(-\theta))}}{2} \left( \Gamma_{TE} \pm \Gamma_{TM} \right)
\end{align*}
\]

Where \(a_{R(\theta)}, a_{R(-\theta)}, a_{L(-\theta)}\) are RHCP antenna and LHCP antenna (denoted as R or L) amplitude pattern responses and angles \(\delta\) and \(\zeta\) are RHCP and LHCP antenna phase pattern responses at given angles of \(\phi\) and \(\pm \theta\).

Since amplitude and phase pattern can be precisely determined using anechoic chamber measurements, the multipath contributions can be minimized by algebraic manipulations (i.e. subtracting and adding two equations in (7)) using following scaling correction:

\[
\text{Scaling Correction}(\phi, \theta) = \frac{a_{R(-\theta)}e^{j[\delta(-\theta) - \phi(-\theta)]}}{a_{L(-\theta)}}
\]

This would require a signal received by the RHCP circuit to be multiplied by the scaling correction and added to the signal received by the RHCP circuit to form a composite signal. Since the correction would be unique for given angles of \(\phi\) and \(\theta\), it would have to be applied separately for each tracked satellite. This should not be a problem, since each satellite has a dedicated channel in the receiver. The cost of implementing this approach is the cost of having a dual receiver in order to process both RHCP and LHCP information. The benefit is the ability to greatly reduce or completely eliminate the effect of multipath.

REFERENCES

[1] [NGS PCO and PCV data base of GNSS antennas: http://www.ngs.noaa.gov/ANTCAL/]
[2] [Geo++ PCO and PCV data base of GNSS antennas: http://gnpcvd.gesopp.de/pcvd/GNPCVDB.html]